Fractions, Decimals and Percentages

Fractions 4

Go deeper investigations

Hilary Koll and Steve Mills

Schofield&Sims

Roof garden design investigation

Objectives

- Understand the role of the numerator and denominator.
- Find fractions of areas.
- Find and recognise equivalent fractions.
- Add fractions with the same denominator.

Introduction

Explain that a building or skyscraper has a flat square roof with an area of 36m^2. The pupils' job is to design a roof garden for it. Tell them that the garden must contain four rectangular sections: in order of increasing size, there should be a small flower-bed (**A**), a pond (**B**), a lawn (**C**) and a paved patio area (**D**). No two sections can be the same size and all must be rectangles. Work through the first examples on the worksheet together.

The maths

The pupils should find the area of each section of the roof as a fraction of the whole roof. Therefore, each section will be a fraction with the denominator 36. If they can, the pupils should then write each of these as a simplified equivalent fraction, as shown in the first example.

Challenge the pupils to come up with as many different designs as they can, in addition to those shown on the worksheet, and build a class list of different solutions, writing addition statements with simplified fractions. Note that some combinations of the garden's features can be arranged in more than one way.

Ask:

- *How many solutions can we find that have the fraction $\frac{1}{36}$?*
- *Is it possible to find a solution where all the fractions have the numerator 1?*
- *Did anyone work systematically to find solutions? Can you explain to us what you did?*

Solutions

There are 21 unique solutions.

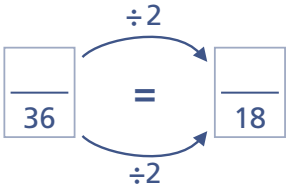
$\frac{1}{36} + \frac{5}{36} + \frac{1}{3} + \frac{1}{2} = 1$
$\frac{1}{36} + \frac{1}{18} + \frac{1}{12} + \frac{5}{6} = 1$
$\frac{1}{36} + \frac{1}{9} + \frac{1}{6} + \frac{25}{36} = 1$
$\frac{1}{18} + \frac{1}{6} + \frac{2}{9} + \frac{5}{9} = 1$
$\frac{1}{18} + \frac{1}{9} + \frac{5}{36} + \frac{25}{36} = 1$
$\frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \frac{5}{12} = 1$
$\frac{1}{9} + \frac{1}{6} + \frac{5}{18} + \frac{4}{9} = 1$

$\frac{1}{36} + \frac{5}{36} + \frac{1}{6} + \frac{2}{3} = 1$
$\frac{1}{36} + \frac{1}{18} + \frac{5}{12} + \frac{1}{2} = 1$
$\frac{1}{18} + \frac{1}{6} + \frac{1}{3} + \frac{4}{9} = 1$
$\frac{1}{18} + \frac{1}{9} + \frac{1}{6} + \frac{2}{3} = 1$
$\frac{1}{18} + \frac{2}{9} + \frac{5}{18} + \frac{4}{9} = 1$
$\frac{1}{12} + \frac{5}{36} + \frac{5}{18} + \frac{1}{2} = 1$
$\frac{1}{9} + \frac{5}{36} + \frac{1}{3} + \frac{5}{12} = 1$

$\frac{1}{36} + \frac{5}{36} + \frac{5}{18} + \frac{5}{9} = 1$
$\frac{1}{36} + \frac{1}{12} + \frac{5}{9} + \frac{1}{3} = 1$
$\frac{1}{18} + \frac{1}{9} + \frac{5}{18} + \frac{5}{9} = 1$
$\frac{1}{18} + \frac{1}{9} + \frac{1}{3} + \frac{1}{2} = 1$
$\frac{1}{12} + \frac{1}{6} + \frac{1}{4} + \frac{1}{2} = 1$
$\frac{1}{9} + \frac{1}{6} + \frac{2}{9} + \frac{1}{2} = 1$
$\frac{5}{36} + \frac{1}{6} + \frac{5}{18} + \frac{5}{12} = 1$

Support

If pupils lack confidence in finding equivalent fractions, draw diagrams like this one to help them. Encourage the pupils to look at the numerator of the fraction they are trying to simplify. Can they divide it by 2, 3, 4, 6, 9, 12 or 18?



Extension

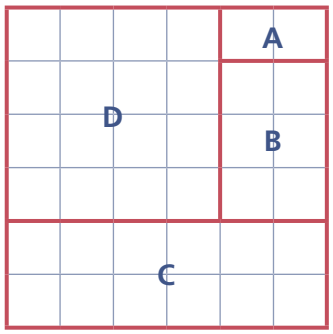
Using squared paper, pupils can investigate other roof-top sizes, such as a rectangle with 5 × 6 or 4 × 6 squares, in the same way. For a further challenge, pupils could include triangular regions where rectangles are split in half.

Roof garden design investigation

Design a roof garden for this building. The roof has an area of 36m². One square on the grid represents 1m². Each garden must have four sections. Each section must be a rectangle. No sections in a garden can be the same size.

smallest **largest**

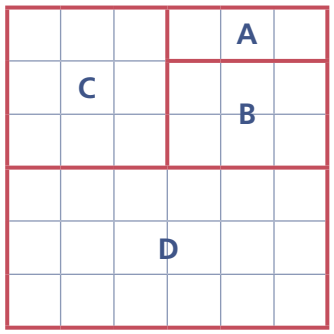
A = flower-bed **B** = pond **C** = lawn **D** = paved patio



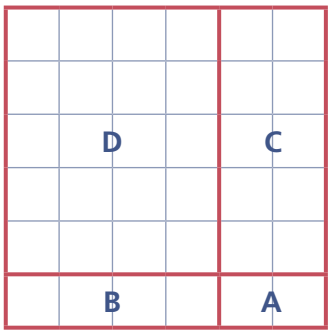
A B C D

$\frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{16}{36} = 1$

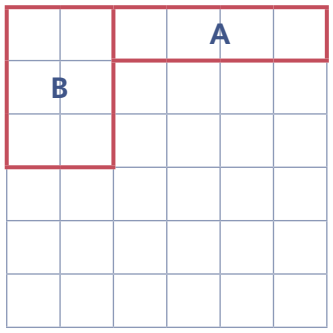
$\frac{1}{18} + \frac{1}{6} + \frac{1}{3} + \frac{4}{9} = 1$



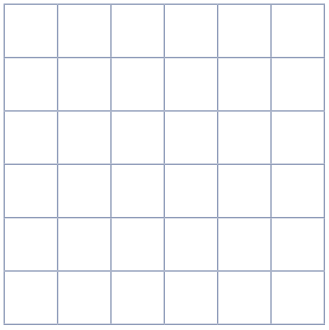
A B C D



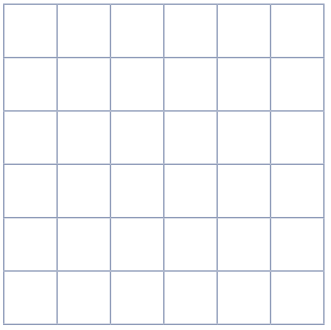
A B C D



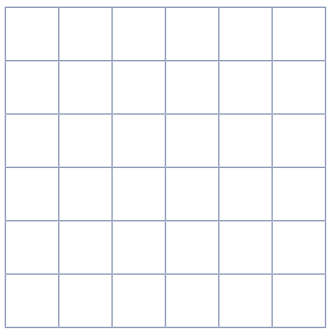
A B C D



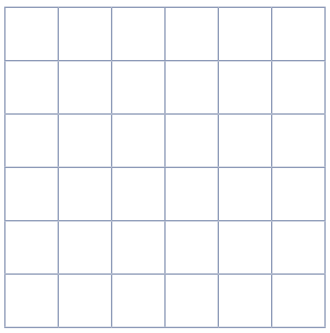
A B C D



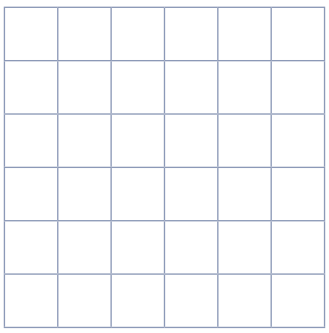
A B C D



A B C D



A B C D



A B C D

A tenths moment

Objectives

- Understand tenths as fractions and decimals.
- Divide one- or two-digit numbers by 10.
- Understand fractions and decimals as the result of division.

Introduction

Explain to the pupils that this activity explores different ways of finding and recording tenths of one- and two-digit numbers, giving answers as fractions and as decimals. Remind the pupils that tenths are the result of division by 10 and show them that the answers can be written as fractions and as decimals. Discuss place value, reminding the pupils that when dividing by 10 the digits of the number move one place to the right. When you give out the worksheet, make sure the pupils understand that they must use each of the digits on the cards to complete the statements on the page.

The maths

As they complete the worksheet, the pupils should discover that each statement will use the same digit, or pair of digits, twice, for example **3** ÷ 10 = $\frac{3}{10}$ and $\frac{1}{10}$ of **23** = **2.3**. This helps them realise that dividing by 10 is easy. Each statement appears on the worksheet three times, but the pupils should complete each one differently. This reminds them that fraction and decimal representations and statements can appear in a variety of ways.

Ask:

- *How do you find one-tenth of a number as a fraction?*
- *How do you find one-tenth of a number as a decimal? Describe what happens to the digits of the number.*

Solutions

Each statement will use the same digit, or pair of digits, twice.
Solutions will be in the form:

6 ÷ 10 = $\frac{\mathbf{6}}{10}$ $\frac{1}{10}$ of **7** = 0.**7**

$\frac{1}{10}$ of **45** = **4.5** **98** ÷ 10 = **9.8**

Support

Give less confident pupils digit cards and a place value grid, so that they can practise moving the digits one place to the right.

Extension

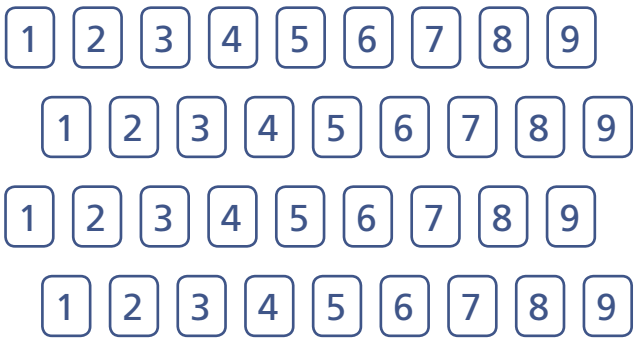
Challenge the pupils to find more than one-tenth of a number, for example $\frac{3}{10}$ of 20 or $\frac{7}{10}$ of 3.

A tenths moment

You have four sets of digit cards showing 1 to 9.

Add one of the digits to each box below to make the statements true.

Cross off the digit cards as you use each one. Make each statement different.



$\square \div 10 = \frac{\square}{10}$	$\frac{1}{10}$ of $\square = 0.\square$
$\frac{1}{10}$ of $\square\square = \square.\square$	$\square\square \div 10 = \square.\square$
$\square \div 10 = \frac{\square}{10}$	$\frac{1}{10}$ of $\square = 0.\square$
$\frac{1}{10}$ of $\square\square = \square.\square$	$\square\square \div 10 = \square.\square$
$\square \div 10 = \frac{\square}{10}$	$\frac{1}{10}$ of $\square = 0.\square$
$\frac{1}{10}$ of $\square\square = \square.\square$	$\square\square \div 10 = \square.\square$

Draw a house

Objectives

- Recognise hundredths as fractions and decimals.
- Divide one- or two-digit numbers by 100.
- Solve problems with measures.

Introduction

Read through the worksheet with the pupils. Explain that, in order to draw the house accurately, each length will need to be one hundred times smaller than the house in real life. To find one-hundredth, each number should be divided by 100. Remind the pupils that when dividing by 100 the digits of the number move two places to the right.

The maths

The pupils will divide one- and two-digit (and, as a challenge, three-digit) numbers by 100 to give the measurements that would be used to draw the house. When dividing 10 by 100, remind them that 0.1 and 0.10 are the same, both showing one tenth and no hundredths. Similarly, when dividing 90 by 100, 0.9 and 0.90 are the same.

Ask:

- *Do you notice what happens to each number when you divide by 100?*
- *What happens to the number if you divide by 10?*
- *Is 0.9 the same as 0.90? Can you explain why?*

Solutions

a)

A	0.07m
B	0.05m
C	0.03m
D	0.02m
E	0.1m or 0.10m
F	0.14m
G	0.9cm or 0.90cm
H	0.35cm
I	2cm or 2.00cm
J	2.5cm or 2.50cm

b) 0.02m

c) 0.9cm

d) 0.14m

Support

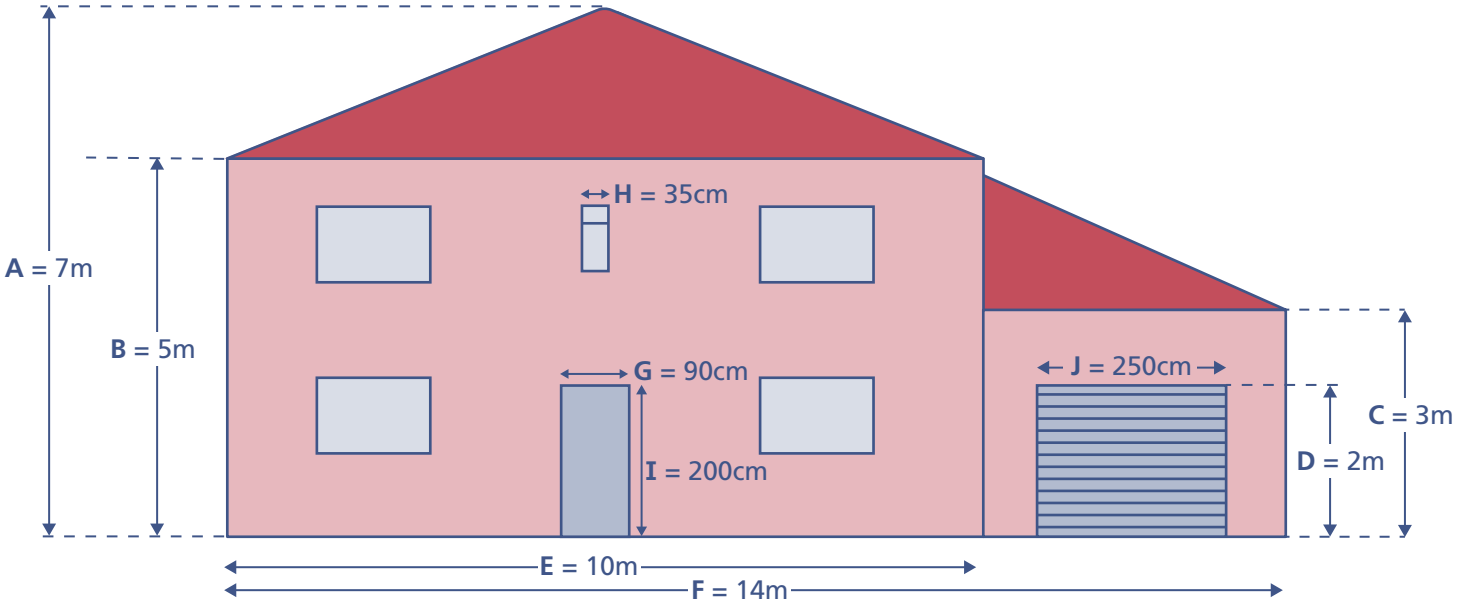
Give less confident pupils digit cards and a place value grid, so that they can practise moving the digits two places to the right.

Extension

Challenge the pupils to draw the house using the measurements they have recorded. They will need to interpret the decimal measurements and convert them from metres to centimetres or from centimetres to millimetres by multiplying by 100 or 10 respectively.

Draw a house

To draw a picture of a house that is one-hundredth of the size of the real house, divide each real-life length by 100. This picture shows the real-life lengths in metres and centimetres.



a) Divide each of the real-life lengths by 100 and complete the table.

	real-life lengths	one-hundredth of the real-life lengths
A	7m	0.07m
B	5m	_____ m
C	3m	_____ m
D	2m	_____ m
E	10m	_____ m
F	14m	_____ m
G	90cm	_____ cm
H	35cm	_____ cm
I	200cm	_____ cm
J	250cm	_____ cm

In a picture that is one-hundredth of the size of the real-life house:

b) how tall is the garage door in metres? _____ m

c) how wide is the front door in centimetres? _____ cm

d) how wide is the whole house and garage together in metres? _____ m